Doppler shift between two rotating disks
by

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If two disks are rotating in opposite directions **v** and **–v**, there will occur a Doppler frequency shift. Thim [1] had found that this shift is equal to his equ. (13), i.e.,

Thim(13): $$f_2 = f_1 \frac{1}{\sqrt{1-(2vc/(v^2+c^2))^2}} = f_1 \frac{1+(v/c)^2}{1-(v/c)^2} \qquad (13)$$

(f_2/f_1) is an additional transverse blue shift between the two disks. This shift would be zero, or (f_2/f_1) = 1, if the two disks would be glued together. Then no frequency shift would occur for a signal emitted form disk1

Thim's two rotating disks are shown below in Fig. (1), where the relevant signal paths are indicated by three red arrows. The corresponding shifts are given by the three equations below:

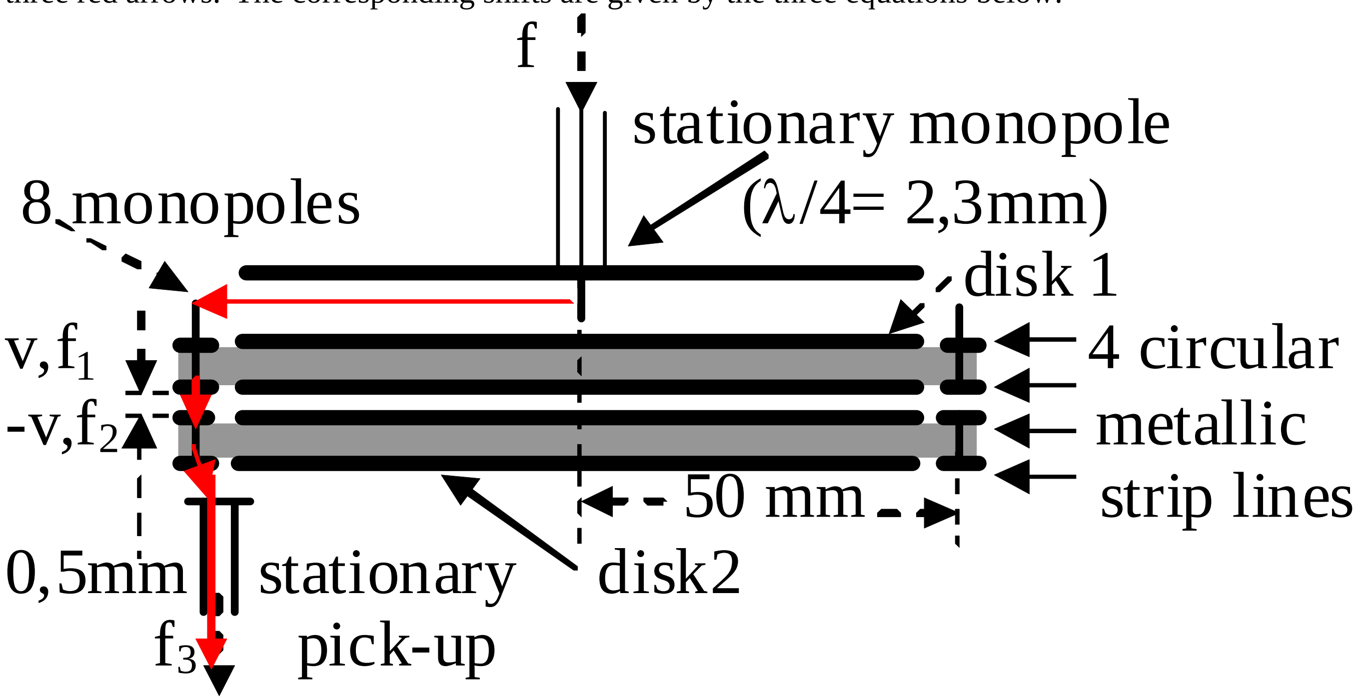

Fig. 1 Two rotating disks

Thim(12): $f_1 = f \frac{1}{\sqrt{1-(v/c)^2}}$ (f_1/f) is a transverse blue shift

Thim(13) $f_2 = f_1 \frac{1}{\sqrt{1-(2vc/(v^2+c^2))^2}} = f_1 \frac{1+(v/c)^2}{1-(v/c)^2}$ (f_2/f_1) is a transverse blue shift between the disks

Thim(14) $f_3 = f_2\sqrt{1-(v/c)^2} = f\ \dfrac{1+(v/c)^2}{1-(v/c)^2}$ (14)

(f_3/f_2) is a transverse red shift, the overall shift for the signal being transmitted along the red colored path i.e. f_3/f is thus a blue shift, equ.(14)

Derivation employing Four-Vectors along all four red colored paths.

$$k_x' = \frac{k_x - \omega v/c^2}{\sqrt{1-(v/c)^2}} \qquad \omega' = \frac{\omega}{\sqrt{1-(v/c)^2}} \tag{K1}$$

$$k_x'' = \frac{k'_x - \omega'(-v)/c^2}{\sqrt{1-(v/c)^2}}, \qquad \omega'' = \frac{\omega' - k_x(-v)}{\sqrt{1-(v/c)^2}} \tag{K2}$$

$$k_x''' = \frac{k_x'' - v\omega''/c^2}{\sqrt{1-(v/c)^2}}, \quad \omega''' = \frac{\omega'' + vk_x''}{\sqrt{1-(v/c)^2}} = \omega\frac{1+v^2/c^2}{[1-v^2/c^2]} \tag{K3}$$

$$k_x'''' = \frac{k_x''' - \omega'''v/c^2}{\sqrt{1-v^2/c^2}} = \frac{-2\omega\cdot v/c^2 - \omega\dfrac{1+v^2/c^2}{1-v^2/c^2}}{\sqrt{1-v^2/c^2}} = \frac{-\omega(1+v^2/c^2)}{(1-v^2/c^2)\sqrt{1-v^2/c^2}} \tag{K4}$$

$$\omega'''' = \frac{\omega''' - vk'''_x}{\sqrt{1-v^2/c^2}} = \omega\frac{(1+v^2/c^2)\sqrt{1-v^2/c^2} + (1+v^2/c^2)^2}{(1-v^2/c^2)(1-v^2/c^2)} = \omega\frac{(1+v^2/c^2)}{1-v^2/c^2}\frac{\{1+v^2/c^2+\sqrt{1-v^2/c^2}\}}{1-v^2/c^2} \cong \omega\frac{1+v^2/c^2}{1-v^2/c^2} \tag{K5}$$

Discussion and Conclusion

Equation (K5) is more complicated than the published equ (14). Interestingly it is very close to (14) as it contains the factor $(1+v^2/c^2)/(1-v^2/c^2)$.

So Thim's equations (12)-(14) are in good accordance with special relativity. In special relativity it is assumed that a Doppler shift occurs between any two moving objects being in relative motion with respect to each other. For an observer on disk 1 disk 1 can be considered being at rest. Then the counter-rotating disk 2 is moving at twice the speed wrt disk 1. If the relativistic theorem of adding velocities is applied correctly one obtains equ.(13). Since there is definitely relative motion between the disks, a transverse shift must result.

Otherwise the shifts (12) and (14) also containing the gamma-factor would not be correct.

Interestingly equation (14) (also K5) has also been derived by Einstein [2] for a moving object when observed under an angle of 90°.

Special relativity thus predicts an overall transverse frequency shift which has not been observed experimentally as reported by Thim [1].

References

[1] H.Thim, "Absence of the Relativistic Transverse Doppler Shift at Microwave Frequencies", *IEEE Trans.Instr.Meas.*, **52**, no.5, pp.1660-1664, 2003

[2] A. Einstein, "On the Electrodynamics of Moving Bodies", *Annalen der Physik*, 17, pp. 891-921, 1905

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